AMENDMENTS TO THE SPECIFICATION

Page 1, line 18, to Page 2, line 7, replace with the following:

--Many difficulties are encountered in such a magnet design problem, and many solutions have been proposed. The various proposed solutions exhibit one or more disadvantages. For example, the problem of volume minimization of electromagnetic coil systems has been considered previously by Kitamura ~~[1]~~ et al, M. Kitamura, S. Kakukawa, K. Mori, and T. Tominaka, "An optimal design technique for coil configurations in iron-shielded MRI magnets," IEEE Tran. Magn., vol. 30, no. 4, pp. 2352-2355, 1994, ("Kitamura"), and by Xu ~~[2]~~ et al, H. Xu, S. M. Conolly, G. C. Scott, and A. Macovski, "Homogeneous magnet design using linear programming," *IEEE Trans. Magn.,* vol. 36, no. .2, pp. 476-483, 2000 ("Xu"). ~~The numbers in square brackets are references to published papers that are identified in Appendix B annexed herewith. The contents of each of those publications are incorporated herein by reference.~~ For simplicity, when discussion of a particular reference is needed or desirable, it may be identified by the name of the lead author ~~or as, for example, Ref. 1, which will be understood to mean the Kitamura paper, or Ref. 2, which will be understood to mean the Xu paper, and so on~~. The reader is urged if needed to review ~~Ref. 1 and Ref. 2~~ these references for a more complete discussion of the various known methods and their pros and cons and for a more complete understanding of the methods described herein. While the methods disclosed by ~~Refs. 1~~ Kitamura and ~~2~~ Xu have benefits, they also have requirements that are undesirable. For example, the method of ~~Ref. 1~~ Kitamura assumes unidirectional currents in the coils, while the method of ~~Ref. 2~~ Xu requires that the length-to-width ratio of the coils be specified. Therefore, neither approach determines, in general, the minimum volume solution, as is defined in this invention. It is also noted that a variety of procedures for optimizing electromagnets have been described that utilize criteria other than coil volume minimization; for a review of these the reader is referred to ~~Ref. 2~~ Xu. --

Page 3, lines 11-21, replace with the following:

-- As with those of ~~Refs. 1~~ Kitamura and ~~2~~ Xu, the method described here employs linear programming (LP) computation using a computer. However, a straightforward application of linear programming may not, by itself, be a practical approach for accurately calculating minimum volume coil arrangements. The reason for this is that the memory requirements and computational time for

a linear programming computation increase rapidly as one increases the number of numerical grid points used to represent the feasible volume. See the discussion in ~~Ref. 2~~ Xu. To circumvent this problem, I first use linear programming with a sparse grid to obtain a coarse grained solution. This solution is then refined by numerically solving with a computer a set of nonlinear equations. The solving of these equations on a dense grid is feasible, because the memory requirements and computational time increase only linearly with the number of grid points. --

Page 5, lines 3-10, replace with the following:

-- I will first define in Section II a statement of the problem in appropriate mathematical terms; I will next provide in Section III the mathematical background for one skilled in this art to understand my new method; I will then describe in Section IV my method for minimizing coil volume including various optional steps for certain purposes; I will then describe in Section V my method for minimizing power; next; I will describe in Sections VI and VII some examples and results in comparison with other proposed solutions and some remarks; and finish in Section VIII with some summarizing comments. ~~Appendices A, B, and C follow~~ Following Section VIII are Section IX, labeled Mathematical Proofs, Section X, labeled Tables I, II, and III, and finally the claims and Abstract.--

Page 6, line 12, replace with the following:

-- The magnetic field generated by the magnet is given by ~~[3]~~ D. B. Montgomery, *Solenoid Magnet Design*. New York: Wiley, 1969 ("Montgomery") --

Page 8, last two lines, replace with the following:

-- The method for determining the current density *J* is founded on three principal mathematical theorems. Here I state these theorems, leaving their proofs for ~~Appendix A~~ Section IX below.--

Page 9, lines 12-14, replace with the following:

-- I note that this formulation has some similarities to, but is not identical to, ones presented by Kitamura ~~et al. [1]~~ and by Xu ~~et a1. [2]~~. For instance, J is assumed to be nonnegative in ~~Ref. 1~~ Kitamura, while the condition of Eq. (12b) is not utilized is ~~Ref. 2~~ Xu. --

Page 10, lines 7-15, replace with the following:

-- This theorem shows how the minimization of V with equality constraints can be reduced to solving a system of nonlinear equations for the λj. These equations are obtained (as is discussed below) by combining Eqs. (13) and (14). A somewhat related result has been previously derived by me for permanent magnet structures ~~[6]~~ , J. H. Jensen, "Optimization method for permanent-magnet structures," IEEE Trans. Magn., vol. 35, no. 6, pp. 4465-4472, 1999 ("Jensen"). However, the equations and assumptions of that earlier development for a permanent magnet structure are not applicable to the electrical coil arrangements which can be designed using the methods and concepts of the present invention which are not disclosed in ~~Ref.[6]~~ Jensen. A useful corollary of the present invention is that a solution with $\lambda_0 = 0$ must have J_0 equal to the smallest current density magnitude that is consistent with the conditions of Eqs. (8) and (13). --

Page 11, line 9, to Page 12, line 5, replace with the following:

-- With the introduction of the numerical grid, the form of the problem given by Theorem 1 becomes a standard linear programming problem as described in ~~Refs. 4 and 5~~ G. Hämmerlin and K.-H. Hoffman, Numerical Mathematics. New York: Springer-Verlag, 1991 ("Hammerlin");and W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery , Numerical Recipes in C: The Art of Scientific Computing. New York: Cambridge University Press, 1992 ("Press"). The functions J and U may be regarded as N-dimensional vectors and the optimization problem has 2N degrees of freedom. The total number of inequality constraints, corresponding to Eqs. (5), (6), and (12), is $4N + 2M_h + 4M_f$.

A variety of algorithms have been proposed for solving linear programming problems, with the most popular being the simplex method ~~[4,5]~~ Hammerlin, Press. All of these are usable in my invention, but I prefer to use a double precision version of a simplex method subroutine given in ~~Ref. 5~~ Press and have found it to be effective when N is a few hundred or less. For larger values of N, three problems arise. First, the method requires sufficient memory for a matrix with approximately $8N^2$ elements or about $64N^2$ bytes. On many computers, this limits N to no more than a few thousand. Second, if ε is small, as is the case when the homogeneous region is required to have a high degree of uniformity, then numerical rounding errors become important. In practice, I have found that this often restricts N to be less than one thousand. Finally, the average computational time scales roughly as N^3, which again can make large values of N impractical.

To be sure, these difficulties are to some extent algorithm dependent. In principle, both the memory and rounding problems could be largely circumvented, but at the price of a longer computational time. However, no linear programming algorithm has been proposed with a computational time that, on average, grows substantially more slowly with N than the simplex method ~~[4]~~ Hammerlin. Thus, a straightforward application of linear programming to the determination of the ideal current density is likely to be limited to sparse grids.--

Page 14, lines 3-5, replace with the following:

-- There are a number of known standard numerical methods for solving equations of the form of Eq. (25) (See ~~Ref. [5]~~ Press). All of these can be used in accordance with my invention. However, for this particular problem, I have found an iterative approach, similar to one described in ~~Ref. [6]~~ Jensen, to be especially effective. In order to employ this method, Eq. (25) must be recast into a different form. Let me define an (M + 1) x (M + 1) matrix--

Page 16, last line to Page 17, line 2, replace with the following:

-- This follows from inspection of Eq. (14). The current density is then found by repeating Step 3, as described above. (A more detailed discussion of the imposition of auxiliary conditions is given in ~~Ref. 6~~ Jensen.) --

Page 18, line 13, to Page 19, line 6, replace with the following:

-- In order to illustrate the optimization method, I apply it to two different examples. First I compute the minimum volume coil arrangement for a magnet that is comparable to a design proposed in the seminal work ~~Garrett [7]~~ M. W. Garrett, "Thick cylindrical coil systems for strong magnetic fields with field or gradient homogeneities of the 6th to 20th order," J. Appl. Phys., vol. 38, no. 6, pp. 2563-2586, 1967 ("Garrett"). In this paper, Garrett gives numerous coil arrangements that generate uniform fields. However, these arrangements are not optimized in any systematic manner. I choose one simple example and demonstrate how my method can lead to an arrangement with the same external geometry, current density magnitude, field strength and homogeneous volume, but with a significantly smaller coil volume. I also consider varying the current density magnitude and show how to minimize the power consumption for resistive coils. As a second example, I minimize the coil volume for a magnet similar to conventional actively shielded superconducting magnets used for whole-body MRI.

The arrangements considered are all symmetrical with respect to the z = 0 plane. As a consequence, the expansion coefficients, a_j and b_j, vanish automatically for even j. The constraints corresponding to these even order coefficients may then be excluded from the nonlinear system, and all the λ_j with even j, except λ_0 must be zero. While this symmetry simplifies the calculation of the ideal current density, it should be emphasized that it is not a requirement for the application of my method. --

Page 19, line 8-16, replace with the following:

-- Figure 3a shows a three coil arrangement 12, 14, 16, taken from Fig. 3 and Table XVI of the work of Garrett ~~[7]~~. The total length of the magnet is 2.8839 ι_0, the inner radius is 0.75 ι_0, and the outer radius is 1.25 ι_0 , where ι_0 is an arbitrary length scale. The total volume of the coils is V = 5.0813 ι_0^3. The magnitude of the current density is $J_0 = 4.9086\ B_0/(\mu_0\iota_0)$, where B_0 is the desired field strength, and the current flows in the same direction in all of the coils. The homogeneous volume with a 20 ppm peak-to-peak field variation is approximately an oblate spheroid with major and minor semiaxes of 0.42 ι_0 and 0.36 ι_0. The coil arrangement is designed to cancel the harmonics of Eq. (16) for $2 \leq j \leq 8$. --

Page 20, lines 5-12, replace with the following:

-- To obtain a design with rectangular coils, I follow step 4 of the optimization procedure and partition the domain D as indicated in Fig. 4a by the 13 rectangles shown. This leads to the coil arrangement shown in Fig. 4b 20-40. In order to obtain a high degree of accuracy, I used N = 70, 000. The volume of the coils is V = 4.62 ι_0^3, which is only slightly larger than that of the ideal solution. The design of Fig. 4b consists of 13 coils. Each coil may be specified by giving the coordinates of the point lying nearest to the origin, (r_{in}, z_{in}), and the coordinates of the point lying farthest from the origin, (r_{out}, z_{out}). These values are listed in Table I ~~of Appendix C~~ below for the coils that lie in the positive z half-plane.--

Page 20, last line, to Page 21, line 6, replace with the following:

-- I now consider optimizing a magnet 110 with an inner radius of 50 cm, an outer radius of 75 cm, and a total length of 125 cm as shown by the rectangle of Fig. 7b. I set B_0 = 1.0 T and require

the homogeneous volume to have a diameter of 45 cm, with a peak-to-peak field variation of less than 20 ppm. The magnitude of the fringe field 112 is restricted to be less than 0.5 mT (5 G) for distances greater than 400 cm from the center of magnet. These specifications are typical for a compact superconducting magnet designed for medical MRI, ~~[8]~~ S. Crazier and D. M. Doddrell, "Compact MRI magnet design by stochastic optimization," J. Magn. Reson., vol. 127, pp. 233-237, 1997 ("Crazier"). --.

Page 29, top line, replace with the following:

-- ~~APPENDIX A~~ Section IX-Mathematical Proofs --

Page 30, 6th line from bottom, to Page 31, line 2, replace with the following:

-- The function on the right side of Eq. (A10) is analytic in the variable z. A standard theorem of analytic functions then indicates that if Eq. (A10) holds for a dense set of z values, then it must hold for all z values ~~[9]~~ ,R. V. Churchhill, J. W. Brown, and R. F. Verhey, *Complex Variables and Applications*. New York: McGraw-Hill, 1974. pp. 286-287 ("Churchill"). However, with the aid of Eqs. (3), (4), and (A2), one can show that the right side of Eq. (A10) vanishes as $z \to \infty$. I then conclude that Eq. (A10), and hence Eq. (A9), cannot be satisfied on a dense set of points, and therefore the volume associated with D_b must be zero. From this it follows that $D_a = D$ (except possibly for a set points of zero measure) and that the condition of Eq. (12b) may be replaced with the stricter condition of Eq. (8). --

Page 35, Delete the entire page.

Page 36, top line, replace with the following:

-- ~~APPENDIX C~~ SECTION X --.